



COSMIC STRINGS

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ABSTRACT

Cosmic strings are linear topological defects that are predicted by some grand unified theories to form during a spontaneous symmetry breaking phase transition in the early universe. They are the basis for the only theories of galaxy formation aside from quantum fluctuations from inflation that are based on fundamental physics. In contrast to inflation, they can also be observed directly through gravitational lensing and their characteristic microwave background anisotropy. It has recently been discovered by F. Bouchet and myself that details of cosmic string evolution are very different from the so-called "standard model" that has been assumed in most of the string induced galaxy formation calculations. Therefore, the details of galaxy formation in the cosmic string models are currently very uncertain.



Introduction

The development of grand unified theories in recent years has generated a wealth of new ideas to be exploited by cosmologists. One of the most exciting developments has been proposal of two separate mechanisms to generate the primordial density fluctuations: quantum fluctuations from inflation and cosmic strings.¹ Inflation induced adiabatic fluctuations have been investigated in great detail. Many variations of this model have been ruled out, but the version with cold dark matter and biasing (i.e. galaxies only form at high peaks in $\delta\rho/\rho$) has been fairly successful.² This model is currently quite popular, but recent observations indicating large scale bulk motions³ and very early galaxy formation^{4,5} seem to contradict the predictions of this theory.

In contrast to the adiabatic fluctuations produced by inflation, the study of gravitational accretion about cosmic string induced fluctuations is still in its infancy. (I will not discuss the recent proposal by Ostriker, Thompson, and Witten⁶ that superconducting cosmic strings might be the seeds for explosive galaxy formation.) Some

preliminary work has been done, but it has not been sufficient to rule out either hot dark matter,^{7,8} cold dark matter,⁹ or just plain baryons in combination with strings. In fact, most of this work has been based on the heuristic “standard model” for string evolution¹⁰ which is (as we shall see) inconsistent with the most recent results from numerical simulations of cosmic string evolution.¹¹ In fact, it is beginning to seem that reliable predictions for galaxy formation with cosmic strings will only be possible when cosmic string evolution codes are combined with cosmological n-body codes.

A major distinction between cosmic strings and inflation is that cosmic strings, if they exist with sufficient mass to produce galaxies, should be directly observable. Cosmic strings should produce detectable anisotropies in the microwave background radiation, and should upon occasion serve as a gravitational lens for distant galaxies or quasars. Furthermore, there are particular details of both the microwave background anisotropy and gravitational lensing that are unique to strings. These “stringy” signature should allow strings to be discovered if they exist or be ruled out if they don’t.

I will now proceed to review the properties of cosmic strings that are of interest to astrophysicists. A discussion of the formation of topological defects via the Kibble mechanism has been given in Dr. Kolb’s lecture. The most important implication of the Kibble mechanism is that a string producing phase transition will produce a large density of infinitely long strings.¹² The simplest theory with cosmic strings is the abelian Higgs model described by the Lagrangian

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{1}{2}|(\partial_\mu - ieA_\mu)\phi|^2 + \frac{1}{2}m^2\phi^*\phi - \frac{\lambda}{4}(\phi^*\phi)^2 \quad (1)$$

The solution to (1) describing a straight cosmic string in the z direction has the form: $\phi = \phi(\rho)e^{i\theta}$ and $A_\theta = A(\rho)$ in cylindrical coordinates (ρ, θ, z) . Since this solution is independent of t and z the solution must be invariant under boosts along the string’s direction (rotations in the $t - z$ plane). This implies that $T_{zz} = -T_{tt}$: the string tension equals the mass per unit length of the string. This means that the characteristic velocity for a curved piece of string is close to the speed of light.

Since string loops oscillate at relativistic velocities, they are rather efficient radiators of gravitational radiation. The gravitational radiation rate for various string loop trajectories has been calculated to be^{13,14}

$$P = \gamma G\mu^2, \quad (2)$$

where G is Newton’s constant, μ is the string tension, and γ is a constant (typically \approx

50–100) which depends on the loop's shape but not its size. (2) implies that a loop of circumference ℓ_0 will have a lifetime of $\ell_0/(\gamma G\mu) \approx 10^4 \ell_0$ for $G\mu \sim 10^{-6}$ (the preferred value).

The only time when the details of the fundamental field theory play an important role in the astrophysics of cosmic strings is when two strings cross. There are two possibilities: either the strings pass through each other and continue on their way, or they intercommute (break and reconnect the other way). This question can be answered by studying the collision of two strings as a problem in classical field theory. This question has been investigated numerically by Shellard¹⁵ and Matzner¹⁶ for the simplest field theories with strings. They have found that intercommutation occurs in almost in every case. Only when the relative velocity between the strings is very large is there some chance that intercommutation will not occur.

The motion of cosmic strings is described by the Nambu Action which leads to the following equation of motion¹⁷ in a Friedman-Robertson-Walker background ($ds^2 = a^2(d\tau^2 - d\mathbf{x}^2)$):

$$\ddot{\mathbf{x}} + 2 \left(\frac{\dot{a}}{a} \right) \dot{\mathbf{x}} (1 - \dot{\mathbf{x}}^2) = \frac{1}{\epsilon} \left(\frac{\mathbf{x}'}{\epsilon} \right)', \quad (3)$$

in the gauge where $\dot{\mathbf{x}} \cdot \mathbf{x}' = 0$ (i.e. the velocity is perpendicular to the string). Dots denote derivatives with respect to conformal time τ , primes denote partial derivatives with respect to the string length parameter σ , and $\epsilon = \sqrt{\mathbf{x}'^2/(1 - \dot{\mathbf{x}}^2)}$ ($E = \mu a \int \epsilon d\sigma$ is thus the string energy). Eq. (3) can be used to determine how the energy density of a cosmic string network will scale with the expansion when string interactions and gravitational radiation are neglected:

$$\dot{\rho}_s = -2 \frac{\dot{a}}{a} (1 + \langle v^2 \rangle) \rho_s, \quad (4)$$

where $\langle v^2 \rangle$ is the mean velocity squared of the string network. Eq. (4) can be understood by examining several special cases: If the strings are straight and stationary, then the string length grows as a while the volume grows as a^3 so that $\rho_s \sim a^{-2}$ which can also be obtained by setting $\langle v^2 \rangle = 0$ in eq. (4). Similarly, if $\langle v^2 \rangle = 1$, then (4) implies that $\rho_s \sim a^{-4}$ just as we expect for ultrarelativistic matter. For a realistic string network, we expect that $\langle v^2 \rangle \approx \frac{1}{2}$ (this is an equality in minkowski space) so that $\rho_s \sim a^{-3}$. If this were still true when the effects of string interactions and gravitational radiation are included, then ρ_s would soon come to dominate over the radiation density ending the radiation dominated era prematurely.

Instead of string domination, the combined effects of string interactions and gravitational radiation is supposed to allow string evolution to be described by a “scaling solution” in which $\rho_s \sim \mu/t^2$ ($\sim 1/a^4$ in the radiation dominated era and $\sim 1/a^3$ in the matter era). The only scale in such a scaling solution is the horizon size, so the string network would look the same at any time during its evolution as long it is measured in horizon sized units.

Observable Consequences of Strings

The most interesting and unique of the observable effects of cosmic strings are the result of their peculiar gravitational field. The space-time in a universe containing only a single straight, static string is locally flat everywhere outside the string, but the effect of the high curvature inside the string is to make the space conical.^{18,19} A circle drawn around the string will have a circumference of $(2\pi - 8\pi G\mu) \times r$ ($8\pi G\mu \simeq 5$ arc seconds for $G\mu \simeq 10^{-6}$). Figure 1 shows the geometry of the 2 spacial dimensions transverse to the string. The wedge is to be discarded and its boundaries are to be identified so that the 2 transverse dimensions form a cone.

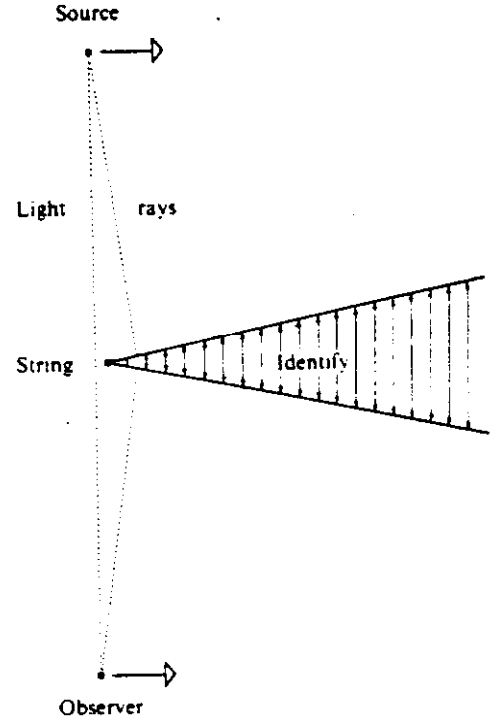


Figure 1

Fig. 1 shows how a cosmic string can serve as a gravitational lens with a light ray passing through flat space on either side of the string. If the string is very straight on the scale of the impact parameters of the light rays, then the conical spacetime is a good approximation to a curved string. In this case, we expect two identical images with a separation of $\sim 4\pi G\mu \approx 2.5$ arc seconds (5 arc seconds times typical geometric and relativistic factors). In the case of extended sources such as galaxies, it is possible that only a fraction of the source will be lensed so that the images are not exactly identical. In this case, the “fractional” image would have a sharp boundary at the string location. Because strings are extended objects, we would also expect to see strong correlations between the positions of lensed objects. In fact, with very deep plates in which the density of galaxies is very high, it might well be possible to trace out the position of a string from the pattern of gravitational lensing.

Recently, Cowie and Hu have discovered a lens candidate²⁰ that meets many of

the “stringy” criteria mentioned above. It is a group of 4-5 double galaxies within a 30” by 30” area on the sky. The members of each pair of images seem to be virtually identical, and their separations are all between 2” and 2.5”. One possible difficulty with the cosmic string interpretation of this object is that the pairs seem to be badly misaligned so that a rather convoluted loop of string would be required to do the lensing. However, it is known that string loops should not be very smooth, but whether they tend to be convoluted enough to account for Cowie and Hu’s object is unclear. There are several observations that are planned for the near future in order to test the string interpretation of this object. David Koo and I have proposed to take very much deeper images of the same field in order to look for more distant galaxies or galaxy pairs in the field. In principle with a deep enough image, we should be able find enough unpaired galaxies between the pairs to rule out the cosmic string interpretation, or find enough pairs to be able to trace out the string profile. In practice, this depends on how well we can identify faint galaxy pairs. Cowie and Hu are planning to get better spectra of some of the pairs in order to determine the velocity splitting between the pairs. For physical pairs, we should expect the velocity splitting to be $\gtrsim 100 \text{ km/sec}$ (the typical orbital speed of galaxies) while the velocity splitting for string lensed pairs should be $\sim 4\pi G/\mu c \lesssim 10 \text{ km/sec}$. Unfortunately, the rotational velocity in a spiral galaxy is also $\gtrsim 100 \text{ km/sec}$, so if the slits of the spectrograph are not properly aligned, a false velocity splitting of $\sim 100 \text{ km/sec}$ may be observed. This situation would be greatly improved if the observation could be done from the space telescope.

Another interesting effect of the string’s conical spacetime can be seen if the string is moving (or if the light source and the observer are moving in the string’s rest frame). With the source and observer moving as shown in Fig. 1, it is apparent that the length of the light path on the right is decreasing with time. This is because the distance between the source (or observer) and the surface of the excluded wedge is decreasing. This causes the right hand light ray to be blue-shifted with respect to the left-hand one. This is what can give rise to the possible $\lesssim 10 \text{ km/sec}$ velocity difference between two images lensed by a string. A more interesting consequence is that it can induce temperature discontinuities²¹ of order $\Delta T/T \sim 8\pi(v/c)G\mu \sim 10^{-5}$ in the microwave background radiation pattern.

Fig. 2 shows the $\Delta T/T$ pattern generated by strings from a simulation of string evolution by Francois Bouchet and myself. (The $\Delta T/T$ calculation was done in collaboration with Albert Stebbins.) The angular scale of the temperature map is $4.4^\circ \times \sqrt{1000/(1+z_{ls})}$ where z_{ls} is the redshift of the surface of last scattering. $z_{ls} \approx 1000$ if

the universe does not become reionized after recombination, but z_{ls} could be as small as 30 with reionization. Fig. 3 shows the strings that are responsible for the MBR fluctuations in Fig. 2 i.e. all those on the past light cone of the observer. Careful comparison of Figs. 2 and 3 will reveal that the sharp discontinuities in Fig. 2 all correspond to string positions in Fig. 3. The converse of this last statement is false, however, because some of the strings in Fig. 3 have little or no transverse motion and therefore do not contribute to the anisotropy. The rms of $\Delta T/T$ in Fig. 2 is $17G\mu \sim 2 \times 10^{-5}$ which is not far below current upper limits on $\Delta T/T$. (The implied upper limit²² on $G\mu$ is 5×10^{-6} .) Therefore it seems quite likely that the next generation of MBR anisotropy experiments should be able to detect the anisotropy due to strings or put stringent upper limits on the string parameter $G\mu$. Once MBR anisotropies have been discovered, it will be possible to check for the “stringy” features of the anisotropy pattern: sharp jumps and

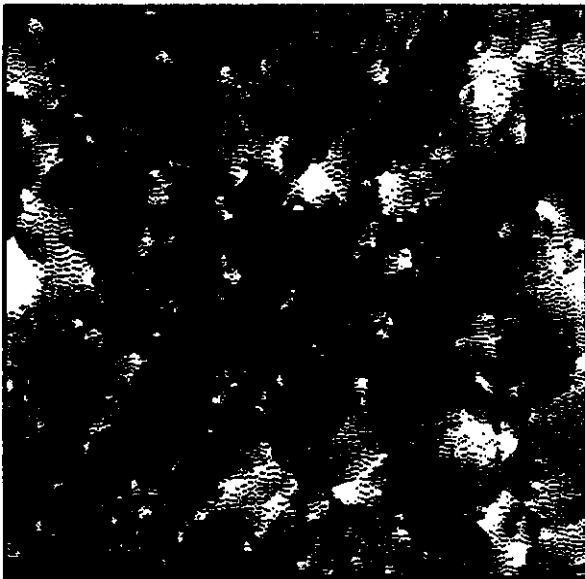


Figure 2

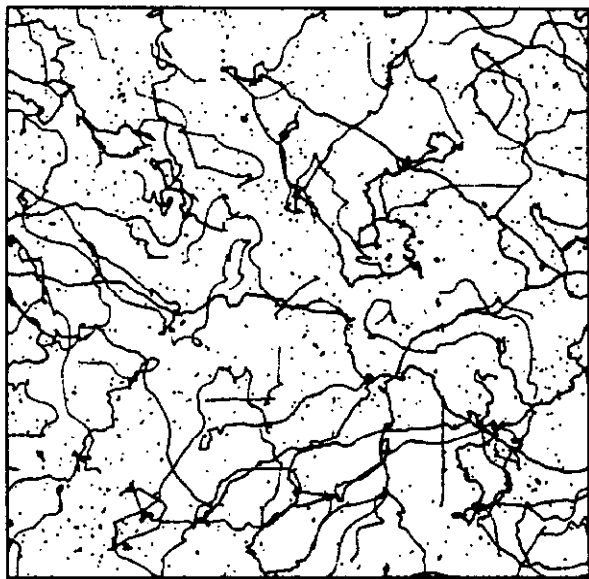


Figure 3

broad plateaus in $\Delta T/T$. This will, of course, require more sensitive detectors than those required to be able to see the anisotropies.

Other observable effects of cosmic strings involve the gravitational radiation coming from cosmic string loops. Primordial nucleosynthesis is sensitive to the total energy density in gravitational radiation in much the same way it is sensitive to the number of neutrino species. Gravitational radiation with periods of order one year will also act as a source of noise for the timing of the millisecond pulsar by varying the distance between the pulsar and earth.²⁵ These two effects are supposed to lead to limits on $G\mu$ that are competitive with the MBR limit of $G\mu < 5 \times 10^{-6}$ given above, but this is not really the case because they depend on the distribution of and radiation rate from small string

loops. Both of these are presently very uncertain. The MBR limit, on the other hand, depends only on the configuration of long strings which is known fairly well. Another drawback of gravitational radiation is that it provides no “stringy” signature in contrast to MBR anisotropies and gravitational lensing.

Perhaps the most interesting consequence of cosmic strings is that they may have been responsible for the formation of galaxies and large scale structure. Before we can discuss this further, however, we must turn to the study of string evolution through numerical simulations because the details of string evolution have important implications for galaxy formation.

Numerical Simulation of Cosmic String Evolution

The central concept in the study of string evolution is the concept of a “scaling solution.”¹⁷ The cosmic string theories for galaxy formation are all built around the idea of a scaling solution, and they would fail if string evolution differed appreciably from scaling. From a practical point of view, it would certainly be impossible to follow string evolution with a numerical simulation from the time when friction from the surrounding gas becomes negligible ($t \sim 10^{-31}$ sec) until the present if the evolution was not scale invariant. The most compelling reason why a scaling solution is necessary is that analytic treatments of string evolution have indicated that the only alternative to the scaling solution in a radiation dominated universe is for ρ_s/ρ_{rad} to grow with time. This will inevitably lead to a universe dominated by strings and their gravitational radiation unless $G\mu \ll 10^{-6}$.

Because all interest in cosmic strings rests on the existence of a scaling solution, the primary aim of numerical simulations is to determine whether a scaling solution actually exists. Once a scaling solution has been established, the simulations can then be used to study the details of the scaling solution because many of the observational effects of strings (particularly galaxy formation) depend on these details.

Both the original cosmic string evolution code of Albrecht and Turok²⁶ and the most recent one developed by Bouchet and myself¹¹ follow the same general procedure. The initial conditions are generated following the procedure introduced by Vachaspati and Vilenkin.¹² First, we randomly assign phases to the Higgs field (ϕ) at the vertices of a cubic lattice. The phases are assumed to vary in the minimal way between lattice sites, and the strings are found by checking for closed circuits on the lattice in which the phase of ϕ varies by $\pm 2\pi$. This gives us an initial string network that consists of 80% “infinite” strings (i.e. those that wrap around our periodic box at least once) and

20% closed loops. On scales much larger than the lattice spacing ξ_0 , our initial string network should closely resemble the real initial state for strings, but on scales $\lesssim \xi_0$ it is probably quite different. Fortunately we need not be concerned about this, because our goal is to test for the scaling solution which should be reached (if it exists at all) from any string configuration with a non-negligible population of infinite strings.

The numerical simulation of string evolution involves two basic processes: a) the motion of strings as described by eq. (3), and b) the detection of string crossing and intercommutation. Taken separately each of these procedures is relatively simple to implement numerically. A modified leapfrog algorithm does quite well in simulating string motion, and it is relatively straight forward to write down a geometric algorithm that will determine when and where two string segments cross. (It is considerably more difficult to make this crossing detection algorithm efficient.)

The major difficulty encountered in the simulations occurs when we attempt to evolve strings that have experienced one or more intercommutations. The problem is that just after a string has undergone an intercommutation, there is a discontinuity in both the tangent to the string (\mathbf{x}') and the string velocity ($\dot{\mathbf{x}}$) at the point of intercommutation. As the string evolves, this discontinuity splits into two discontinuities known as "kinks" which travel in opposite directions along the string. These "kinks" have a very long lifetime so that their decay is virtually undetectable in the short time-scales that are available in numerical simulations. Therefore, it is quite desirable to be able to evolve them properly. Unfortunately, the short wavelength Fourier modes associated with these discontinuities are not simulated very well by most differencing schemes, and it is necessary to use some sort of numerical diffusion in order to damp out short wavelength instabilities. The drawback with this is that the numerical diffusion also tends to damp out the kinks. We have attempted to minimize the numerical diffusion in our evolution scheme by invoking it only when a short wavelength instability starts to develop. This works quite well when the density of kinks is small, but by the end of some of our simulations there is one kink for every 5 string points. Therefore, we are currently developing a new evolution scheme that can handle the kinks quite accurately without any numerical diffusion.

Figure 4 shows a box half a horizon length on a side that was cut out of one of our radiation era string simulations after an expansion by a factor of 3.4 from the initial state. It is immediately clear from Fig. 4 that the majority of string length (actually 75%) is in the form of small loops in contrast to the initial state which had 80% infinite

strings. This is direct evidence that loop production does indeed transfer large amounts of energy from the infinite string network to small loops as is required for a scaling solution. This can be seen more quantitatively in Fig. 5 which shows plots of the string energy density times t^2 as a function of time. In a scaling solution this quantity should be constant. The dashed curve in Fig. 5 shows the total energy density for all the strings in one simulation while the solid curve shows the energy density for only the long ($E > 3.2\mu ct$) strings. Fig. 5 indicates that we have indeed reached a scaling solution for the long strings but not for all the strings in the box. In fact, it is not possible to reach a scaling solution for the smallest loops given the nature of our simulations. It is only when the horizon grows to be fairly large that we even have resolution to study loops that are very small compared to the horizon. Since this occurs near the end of a run, the small loops do not have sufficient time to relax to their scaling solution abundances. Once we do have this resolution, it will still take some time for the loops at this scale to reach scaling. Fortunately, very small loops should have only a small effect on the evolution of the long strings. This means that a scaling solution can indeed be established from this type of simulation.

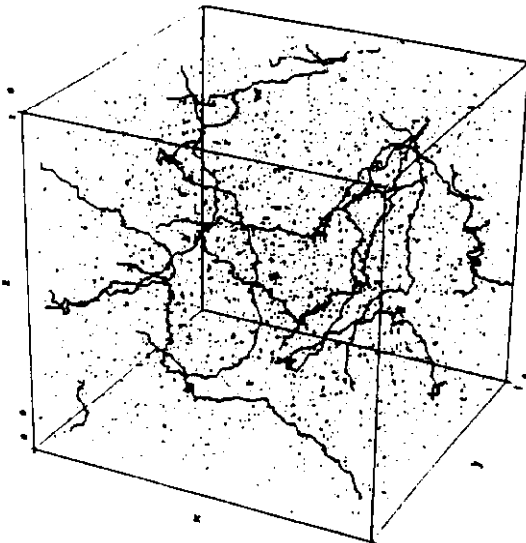


Figure 4

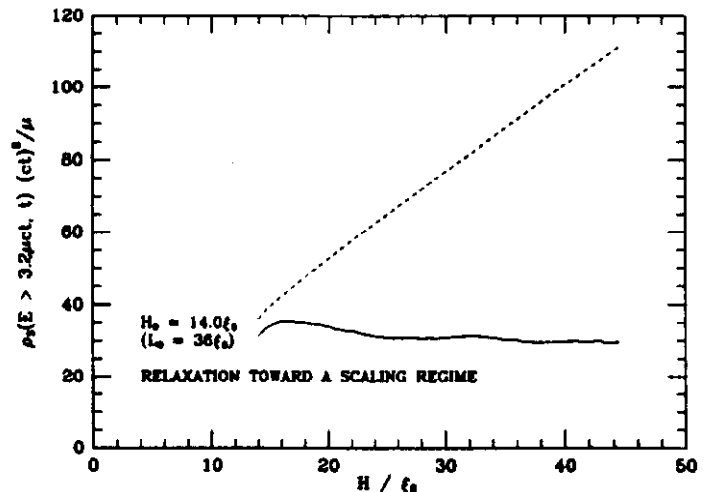


Figure 5

In order to confirm the theoretical prediction that the scaling solution is stable, we have evolved configurations with larger and smaller initial horizon-size H_0 , i.e. with different initial string energy densities. As is shown in Fig. 6, configurations with larger initial energy densities chop off many loops in order to lower the energy density in long strings ρ_{LS} , while in string-poor configurations more string-stretching and lower loop production rate yield an energy density increase. In other words, the scaling solution appears to be a stable fixed point, and perturbed configurations tend to relax to the scaling

solution.

We checked the dependence of these results on our various numerical parameters, in particular the size of the computation box to check for boundary effects, the density of points to determine possible sampling effects, as well as the time step requirements, or the value of our numerical lower cutoff. None of those but the last matters. This cutoff is implemented by requiring a minimal number of points for a loop to be allowed to chop off the network. We ran the simulation corresponding to the bottom curve of Fig. 6 ($H_0 = 8.7\xi_0$) with various cutoffs λ measured in units of the persistence length of the initial lattice (ξ_0). The results are shown in Fig. 7. The solid curves have $\lambda = 1, 0.5, 0.3$, and 0.15 while the dashed curve is for $\lambda = 0.2$. Although the results are somewhat ambiguous, the curves do seem to be converging as $\lambda \rightarrow 0$. Further analysis of these runs reveals that the total energy production in small loops is independent of λ . The difference between these runs is that with small λ , the loops fragment more and the reconnection of the smaller “child” loops is less important. Thus, loop production is more efficient when λ is small. Because we believe that we understand the reason for this cutoff effect, we can extrapolate our results to the physical case ($\lambda = 0$) and determine the scaling solution energy density for long strings: $\rho_{LS} = \zeta_{rad} \mu (ct)^{-2}$, with $\zeta_{rad} = 20 \pm 10$.

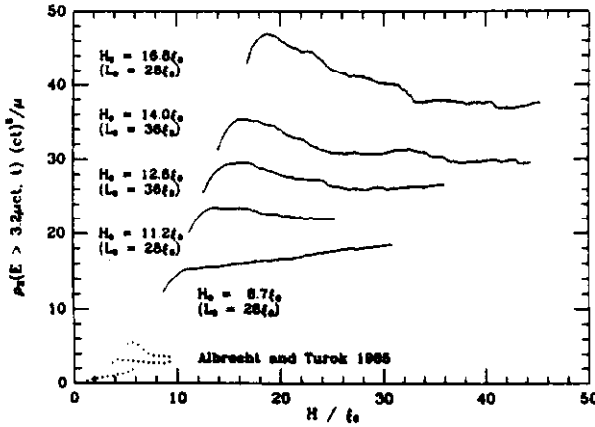


Figure 6

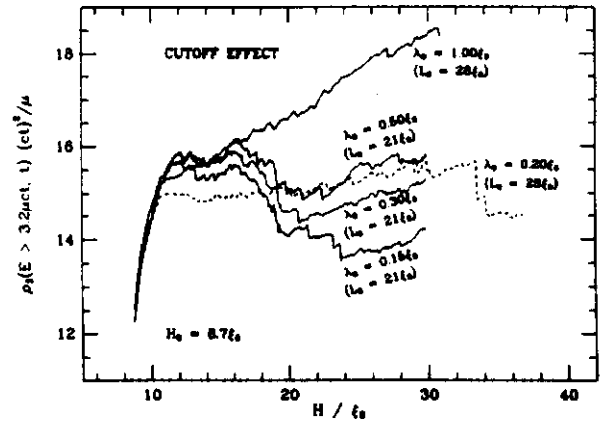


Figure 7

Cosmic String Evolution and Galaxy Formation

The possibility that cosmic strings may serve as seeds for galaxy formation has generated more interest than any of the other observable consequences of strings. Most of the work that has already been done on this subject has assumed the so-called “standard model” of string evolution which holds that:

1. The string network evolves according to a scaling solution with of order one long string crossing each horizon volume.
2. The string network loses energy at a sufficient rate to maintain the scaling solution by chopping off horizon sized loops.
3. These horizon sized "parent loops" typically fragment into 10 smaller "child" loops before stable non-self-intersecting are found.
4. The initial velocities of these noninteracting "child" loops are small enough so that loop motion has little effect on loop correlations.

The numerical simulations of cosmic string evolution by Bouchet and myself¹¹ has indicated that each of these statements is at least partially incorrect. We find that:

1. The string network does indeed evolve according to a scaling solution, but there are roughly 10 or 20 long strings crossing each horizon volume.
2. The string network loses some energy by chopping off horizon sized loops, but at least as much energy is lost by chopping off loops \ll than the horizon size.
3. Those "parent loops" that are horizon sized tend to fragment into $\gg 10$ "child" loops so that virtually all the non-self-intersecting loops are \ll than the horizon.
4. The initial velocities of the noninteracting "child" loops are $\gtrsim 0.5c$ so that loop motion tends to wash out the loop correlations.

The main distinctions between our results and the "standard scenario" can be seen fairly easily from Fig. 4. There are certainly many different long strings crossing this box, and the box is only half a horizon length on a side. Another striking feature of Fig. 4 is that although the strings are reasonably straight on the scale of the box size, they have significant structure on much smaller scales. This short wavelength structure is just the accumulation the kinks which, as mentioned above, are produced in great abundance and decay only very slowly. The short wavelength structure is responsible for the production of very small loops directly from the long strings and for the tendency of the large parent loops to split into very small child loops. As a result the loops in non-self-intersecting trajectories (those that can seed galaxies) are very much smaller than the horizon; there are very few with energy $> 0.1\mu ct$ (radius $> 0.01\mu ct$). This tends to reduce the energy density of the loops with respect to the long strings²³ which implies that the infall of matter into long string wakes²⁷ will be more important than had previously been supposed.

Another significant departure from the standard scenario are the relatively large ($\gtrsim 0.5$) initial center of mass velocities of the loops in our simulations. Velocities of this magnitude are not surprising in view of the fact that $v_{\text{rms}} \simeq 0.7$ for the infinite strings in our simulation. The loops that form as a result of multiple fragmentations of larger parent loops might be expected to have even larger velocities because we expect that these loops will have inherited some of the peculiar motion from each of their ancestors. These relatively large velocities, coupled with the fact that the mean separation of the loops at birth is much smaller than the horizon imply that most of the loops will have moved considerably further than the mean separation in a few expansion times. This has several important implications for the galaxy formation scenario. A standard premise of this scenario has been that each string loop of the appropriate size corresponds to one object: either a galaxy or a cluster. This assumption might still be ok for loops that are formed long before the universe becomes matter dominated, but loops that are massive enough to accrete Abell clusters do not fall into this category.

The remarkable correspondence between the two-point correlation function of Abell clusters and the two-point correlation function for string loops found by Turok²⁸ has long been regarded as the major success of the cosmic string galaxy formation scenario. We have computed the loop two-point correlation function in both the radiation and matter dominated eras, and the results are summarized in Figs. 8 and 9. In each of

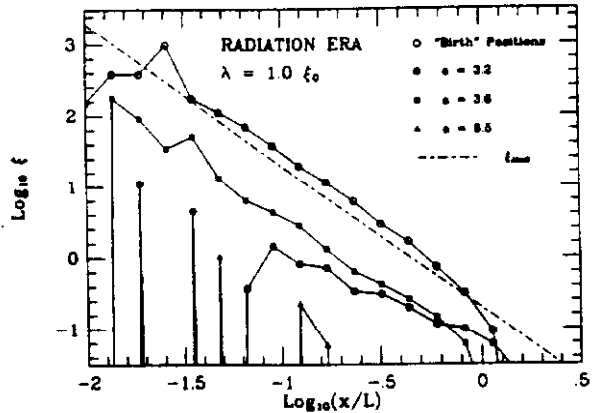


Figure 8

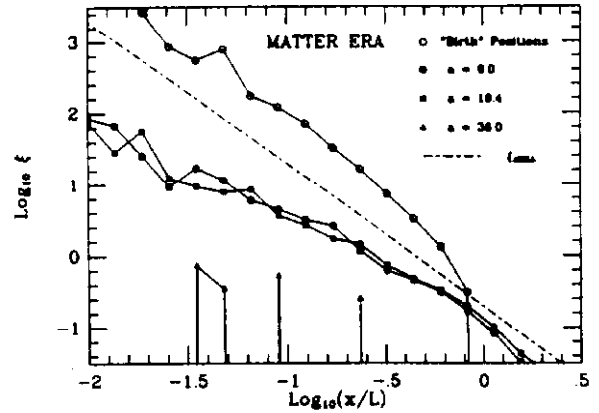


Figure 9

these figures, the open circles correspond to the correlation of loops at birth, i.e. their subsequent displacement was ignored. It is slightly higher than the dashed line which corresponds to an y^{-2} fit to the cluster data and Turok's results, but the slope is virtually the same. When we compute instead the correlations between the loops present at a given time in the simulation box, we obtain the curves labeled by filled circles (at the end of the run) and solid squares (at an intermediate time). For the matter era run,

these correlations are still in reasonable agreement with the observations, but for the radiation era our loop correlations are somewhat lower. The reason for the difference between the “birth” correlation function and the correlation function at fixed time is that much of the loop correlations have been erased by the loop motion since their “birth.” This effect is much more dramatic if we extrapolate the motion of the loops to a time slightly (less than one expansion time) after the end of the run (triangles in Figs. 8 and 9). Now, the correlation function is almost completely washed out. This means that most of the contribution to the correlation function at a fixed time comes from loops that have just been formed.

One possible problem with the correlation functions plotted in Figs. 8 and 9 is that we have used all the loops in the simulation to calculate the correlation function when, in fact, many of the smallest loops may be “unphysical” because they are prevented from fragmenting into still smaller loops only by our limited resolution. To avoid these potentially “unphysical” loops, we have calculated the two point correlation function for only those loops that are much larger than our lower cutoff. The results turn out to be virtually the same except that the statistics are much worse. We also tried lowering the loop center of mass velocities by a factor of two and found that the correlations still tended to be washed out (although slightly slower). We therefore conclude that Figs. 8 and 9 give a reasonably accurate picture of the two-point correlations of string loops.

The implications of these correlations functions for the formation of structure are not entirely clear. The loops that are supposed to be responsible for the formation of Abell clusters are produced early in the matter dominated era. This means that they start accreting matter from the moment of birth. The matter accreted will initially form a long and thin pencil-like object²⁹ stretching from the loops initial position to its final position. The initial positions of the loops would be highly correlated but the final ones would not. Furthermore, the one loop-one object correspondence of the “standard scenario” is probably wrong. For instance, Abell clusters might form at the intersections of loop trajectories, or a single loop might give rise to several clusters in a line. Another complication is that the wakes formed by matter falling in behind the long strings²⁷ are much more important than had previously been thought. (This is because the scaling solution energy density in long strings is much larger than in the “standard scenario.”) The implication of all this is that it will be difficult to tell exactly what type of Abell cluster correlation function will be predicted by cosmic strings without a more detailed calculation. Although the cosmic string scenario’s prediction for the Abell cluster two point correlation function is no longer the clear success that it once seemed, it is clear

that cosmic string loops are highly correlated initially and that Abell cluster correlations are much easier to explain in the cosmic string model than with inflationary adiabatic fluctuations.

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